Označavanje mehaničkih veličina

$\vec{F}$ - sila vektorska veličina koju definiše intenzitet, pravac, smer i napadna tačka

F - intenzitet sile

Projekcije sile u pravcima osa Dekartovog koordinatnog sistema

$F_X = X$,

$F_Y = Y$, Projekcije vektora sile na ose su **skalarne** veličine

$F_Z = Z$.

Projekcije na ravni u Dekartovom koordinatnom sistemu

$\overrightarrow{Fxy}$,

$\overrightarrow{Fyz}$, Projekcije vektora sile na koordinatne ravni su **vektorske** veličine

$\overrightarrow{Fzx}$

Jedinični vektori Dekartovog pravouglog koordinatnog sistema u pravcu x ose $\vec{\imath}$, ose y $\vec{\jmath}$, u pravcu z ose $\vec{k}$. Intenzitet jediničnih vektora je jednak 1.

$$\vec{\imath}\times\vec{\jmath} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}\vec{\imath} - \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}\vec{\jmath} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\vec{k} = \vec{k}$$

$$\vec{\jmath}\times\vec{\imath} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}\vec{\imath} - \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}\vec{\jmath} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}\vec{k} = -\vec{k}$$

$$\vec{\imath}\times\vec{k} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}\vec{\imath} - \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\vec{\jmath} + \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}\vec{k} = -\vec{\jmath}$$

$$\vec{\jmath}\times\vec{k} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\vec{\imath} - \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}\vec{\jmath} + \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}\vec{k} = \vec{\imath}$$

Sila kao vektor u pravouglom koordinatnom sistemu

$\vec{F} = F_x\vec{\imath} + F_y\vec{\jmath} + F_z\vec{k} = X\vec{\imath} + Y\vec{\jmath} + Z\vec{k}$

Rezultantna sila

$\vec{F} = F_{rx}\vec{\imath} + F_{ry}\vec{\jmath} + F_{rz}\vec{k} = X_r\vec{\imath} + Y_r\vec{\jmath} + Z_r\vec{k}$

Projekcije rezultante

$F_{rX} = X_r$,

$F_{rY} = Y_r$,

$F_{rZ} = Z_r$.

Moment sile $\vec{F}$ za tačku O $\vec{M}_O^{\vec{F}}$ je vektor definisan intenzitetom, pravcem smerom i napadnom tačkom.

Moment sile $\vec{F}$ za tačku O $\vec{M}_O^{\vec{F}}$ kao vektor u pravouglom koordinatnom sistemu

$\vec{M}_O^{\vec{F}} = M_{Ox}^{\vec{F}}\,\vec{\imath} + M_{Oy}^{\vec{F}}\vec{\jmath} + M_{Oz}^{\vec{F}}\vec{k}$

$M_{Ox}^{\vec{F}}$ projekcija momenta sile za tačku O $\vec{M}_O^{\vec{F}}$ na osu x

$M_{Oy}^{\vec{F}}$ projekcija momenta sile za tačku O $\vec{M}_O^{\vec{F}}$ na osu y

$M_{Oz}^{\vec{F}}$ projekcija momenta sile za tačku O $\vec{M}_O^{\vec{F}}$ na osu z

VEKTORSKA ALGEBRA

Tačke A (a_X, a_Y, a_Z) i B (b_X, b_Y, b_Z) koordinatni početak O(0, 0, 0)

Vektor $\overrightarrow{AB}$ =(b_X-a_X, b_Y-a_Y, b_Z-a_Z) koordinata druge tačke manje koordinate prve

Vektor $\overrightarrow{OA}$ =(a_X -0, a_Y-0, a_Z-0)= $\vec{a}$

$\vec{a} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}$

$\vec{b} = b_x\vec{\imath} + b_y\vec{\jmath} + b_z\vec{k}$

Brojni primer:

A(1,0,5); B(3,1,2); C(-1,2,4); D(1,3,0)

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ =B-A=(3-1, 1-0, 2-5)=(2,1,-3)

$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ =D-C=(1+1, 3-2, 0-4)=(2,1,-4)

$\overrightarrow{AB} \neq \overrightarrow{CD}$

Zbir vektora $\vec{a} + \vec{b} = \vec{s} = (a_x + b_x)\vec{\imath} + (a_y + b_y)\vec{\jmath} + (a_z + b_z)\vec{k} = s_x\vec{\imath} + s_y\vec{\jmath} + s_z\vec{k}$

Kolinearni vektori $\vec{b} = n\vec{a} \rightarrow b_x = na_x; \quad b_y = na_y; \quad b_z = na_z$

Vektor je apsolutna vrednost vektora pomnožena sa jediničnim vektorom

$\vec{a} = a\,\vec{a}_o \qquad \left|\vec{a}\right| = \sqrt{a_x^2 + a_y^2 + a_z^2} \qquad |\overrightarrow{a_o}| = 1$

Brojni primer:

$\vec{a} = 1\vec{\imath} + 0\vec{\jmath} + 5\vec{k}$ i

$\vec{b} = 3\vec{\imath} + 1\vec{\jmath} + 2\vec{k}$

Zbir vektora

$\vec{a} + \vec{b} = \vec{s} = (1 + 3)\vec{\imath} + (0 + 1)\vec{\jmath} + (5 + 2)\vec{k} = 4\vec{\imath} + 1\vec{\jmath} + 7\vec{k}$

Vektor i jedinični vektor

$$\left|\vec{a}\right| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{1^2 + 0^2 + 5^2} = \sqrt{1 + 0 + 25} = \sqrt{26}$$

$$\vec{a}_0 = \frac{a_x}{|\vec{a}|}\vec{\imath} + \frac{a_y}{|\vec{a}|}\vec{\jmath} + \frac{a_z}{|\vec{a}|}\vec{k} = \frac{1}{\sqrt{26}}\vec{\imath} + \frac{0}{\sqrt{26}}\vec{\jmath} + \frac{5}{\sqrt{26}}\vec{k}$$

Uglovi pravca vektora sa koordinatnim osama

$$\cos\alpha = \frac{a_x}{a};\ \cos\beta = \frac{a_y}{a}\ ;\ \ \cos\gamma = \frac{a_z}{a}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{\left(a_x^2 + a_y^2 + a_z^2\right)}{a^2} = 1$$

$$\vec{a} = a\,\vec{a}_o = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k} = a\left(\vec{\imath}\cos\alpha + \vec{\jmath}\cos\beta + \vec{k}\cos\gamma\right)$$

$$\vec{a}_0 = \vec{\imath}\cos\alpha + \vec{\jmath}\cos\beta + \vec{k}\cos\gamma$$

Uglovi sa koordinatnim osama sa primerom

$$\cos\alpha = \frac{\vec{a}\cdot\vec{\imath}}{|\vec{a}|\cdot|\vec{\imath}|} = \frac{1\cdot1+0\cdot0+5\cdot0}{\sqrt{26}\cdot1} = \frac{1}{\sqrt{26}}$$

$$\cos\beta = \frac{\vec{a}\cdot\vec{\jmath}}{|\vec{a}|\cdot|\vec{\jmath}|} = \frac{1\cdot0+0\cdot1+5\cdot0}{\sqrt{26}\cdot1} = \frac{0}{\sqrt{26}}$$

$$\cos\gamma = \frac{\vec{a}\cdot\vec{k}}{|\vec{a}|\cdot|\vec{k}|} = \frac{1\cdot0+0\cdot0+5\cdot1}{\sqrt{26}\cdot1} = \frac{5}{\sqrt{26}}$$

Za vektor $\vec{b} = 3\vec{\imath} + 1\vec{\jmath} + 2\vec{k}$ uglovi su

$$\left|\vec{b}\right| = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$\cos\alpha_1 = \frac{\vec{b}\cdot\vec{\imath}}{|\vec{b}|\cdot|\vec{\imath}|} = \frac{3\cdot1+1\cdot0+2\cdot0}{\sqrt{14}\cdot1} = \frac{3}{\sqrt{14}}$$

$$\cos\beta_1 = \frac{\vec{b}\cdot\vec{\jmath}}{|\vec{b}|\cdot|\vec{\jmath}|} = \frac{3\cdot0+1\cdot1+2\cdot0}{\sqrt{14}\cdot1} = \frac{3}{\sqrt{14}}$$

$$\cos\gamma_1 = \frac{\vec{b}\cdot\vec{k}}{|\vec{b}|\cdot|\vec{k}|} = \frac{3\cdot0+1\cdot0+2\cdot1}{\sqrt{14}\cdot1} = \frac{2}{\sqrt{14}}$$

Kolinearni vektori

$\vec{d} = 3\vec{b} = 3(3\vec{i} + 1\vec{j} + 2\vec{k}) = 9\vec{i} + 3\vec{j} + 6\vec{k}$

Skalarni proizvod

$(\vec{a} \cdot \vec{b}) = a \cdot b \cdot cos(\vec{a}, \vec{b}) = a_x b_x + a_y b_y + a_z b_z$

$(\vec{a} \cdot \vec{a}) = a^2$;

$(\vec{i} \cdot \vec{i}) = (\vec{j} \cdot \vec{j}) = (\vec{k} \cdot \vec{k}) = 1;\ (\vec{i} \cdot \vec{j}) = (\vec{j} \cdot \vec{k}) = (\vec{k} \cdot \vec{i}) = 0$

$(\vec{i} \cdot \vec{j}) = (\vec{j} \cdot \vec{k}) = (\vec{k} \cdot \vec{i}) = 0$ ose grade uglove od 90^o otuda $\cos 90^o = 0$

Skalarni proizvod i komutacija

$(\vec{a} \cdot \vec{b}) = (\vec{b} \cdot \vec{a})$

Zbir skalarnog proizvoda dva vektora i trećeg vektora

$(\vec{a} \cdot \vec{b} + \vec{c}) = (\vec{a} \cdot \vec{b}) + (\vec{a} \cdot \vec{c})$

Vektorski proizvod

$$\vec{a} \times \vec{b} = [\vec{a}\vec{b}] = \vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = c_x\vec{i} - c_y\vec{j} + c_z\vec{k}$$

$$c_x = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}, \qquad c_y = \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}, \ c_z = \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix},$$

Komutacija kod vektorskog proizvoda

$\vec{a} \times \vec{b} = [\vec{a}\vec{b}] = -\vec{b} \times \vec{a} = -[\vec{b}\vec{a}]$

Koplanarnost

$\vec{a} \times \vec{b} = [\vec{a}\vec{b}] = 0$

$\vec{c} \cdot \vec{a} \times \vec{b} = (\vec{c} \cdot [\vec{a}\vec{b}]) = 0$

Brojni primer:

$\vec{a} = 1\vec{i} + 0\vec{j} + 5\vec{k}$ $\quad\quad$ $\vec{b} = 3\vec{i} + 1\vec{j} + 2\vec{k}$ $\quad\quad$ $\vec{c} = 2\vec{i} + 4\vec{j} + 3\vec{k}$

$$\left|\vec{c} \cdot \vec{a} \times \vec{b}\right| = \begin{vmatrix} c_x & c_y & c_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} 2 & 4 & 3 \\ 1 & 0 & 5 \\ 3 & 1 & 2 \end{vmatrix} =$$

$$= 2(0 \cdot 2 - 5 \cdot 1) - 4(1 \cdot 2 - 3 \cdot 5) + 3(1 \cdot 1 - 3 \cdot 0) = -10 + 52 - 6 = 36$$

Vektori nisu u ravni a zapremina prizme koju obrazuju je V=36

Zapremina prizme $V = \left|\vec{a} \cdot \vec{b} \times \vec{c}\right|$