Zadatak1

grafički rad iz otpornosti materijala

dr Rade Đukić

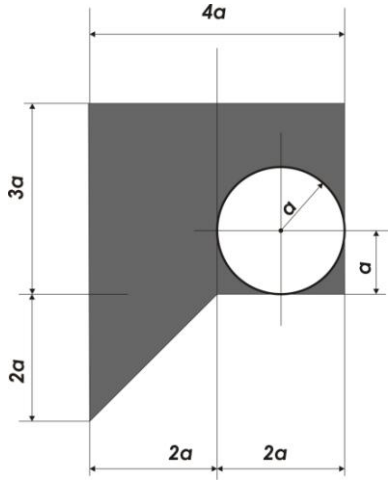
1

Postupak pri određivanju momenata inercije složene površine

1. Izabrati koordinatni sistem Oxy i odrediti položaj težišta
2. Odrediti momente inercija za težišne ose svake površine, pa primenom Štajnerove teoreme odrediti momente inercije za težišne ose složene površine
3. Odrediti ugao glavnih centralnih osa inercije
4. Odrediti glavne centralne momente inercije
5. Odrediti poluprečnike elipse inercije i nacrtati elipsu inercije

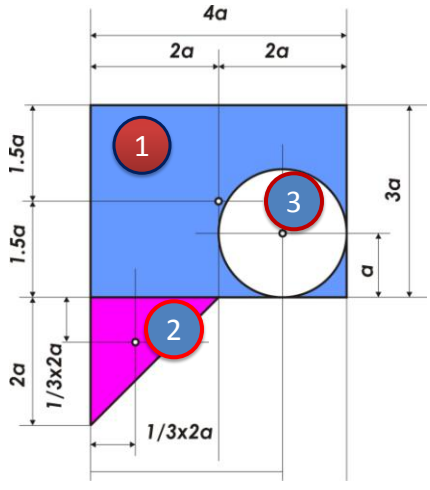
2

analizirati složenu ravnu površinu



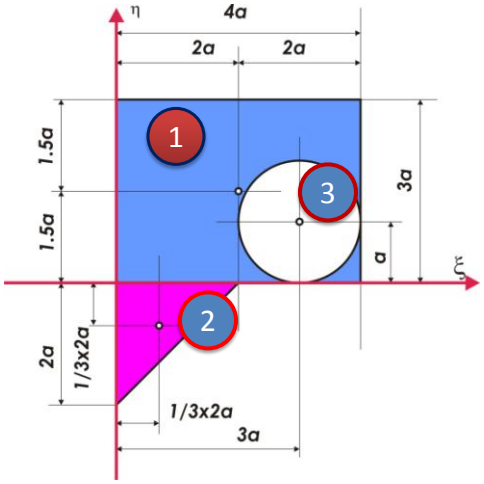
3

podeliti površinu na konačan broj površina sa poznatim površinama i položajima težišta



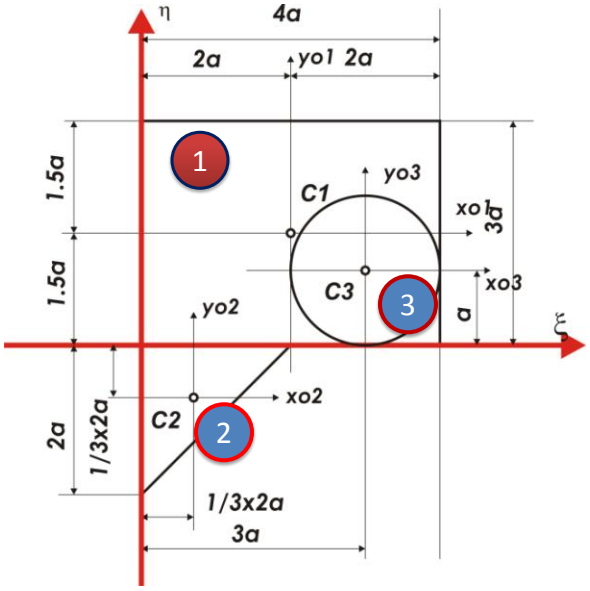
4

Izabrati koordinatni sistem Oxy, sa što povoljnijim položajem težišta i površina prema osama



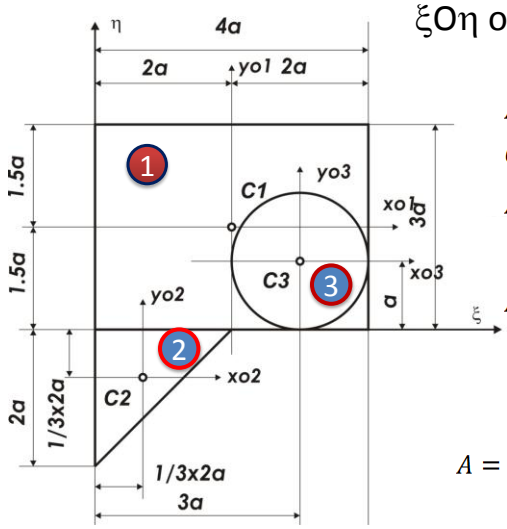
5

u odabranom koordinatnom sistemu ξOη odrediti koordinate težišta površina



6

u odabranom koordinatnom sistemu ξOη odrediti koordinate težišta



$A_1 = 4a \cdot 3a = 12a^2 = 12cm^2,$

$C_1\left(2a, \frac{3a}{2}\right) = (2,1.5)$

$A_2 = \frac{2a \cdot 2a}{2} = 2a^2 = 2cm^2,$

$C_2\left(\frac{2a}{3}; -\frac{2a}{3}\right) = (0.666; -0.666)$

$A_3 = a^2\pi = 3.14\ cm^2,$

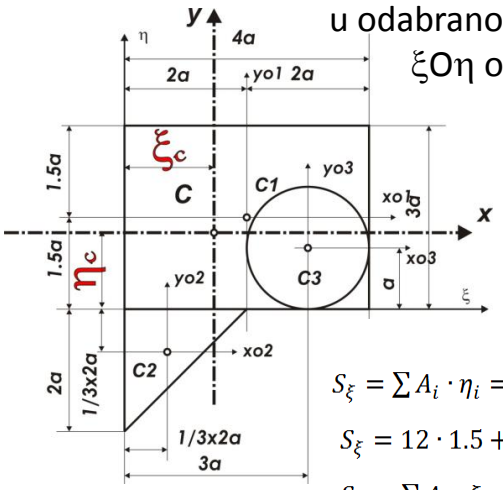
$C_3(3a, a) = (0, -1)$

$A = \sum A = A_1 + A_2 - A_3$

$A = 12a^2 + 2a^2 - 3.14a^2 = 10.858a^2 = 10.86\ cm^2$

7

u odabranom koordinatnom sistemu ξOη odrediti koordinate težišta



$\eta_c = \frac{13.528}{10.86} = 1.245cm$

$\xi_c = \frac{15.912}{10.86} = 1.465\ cm$

$\xi_c = \frac{S_\eta}{A} = 0; \quad \eta_c = \frac{S_\xi}{A} = 0;$

$S_\xi = \sum A_i \cdot \eta_i = A_1 \cdot \eta_1 + A_2 \cdot \eta_2 - A_3 \cdot \eta_3$

$S_\xi = 12 \cdot 1.5 + 2 \cdot (-0.666) - 3.14 \cdot 1 = 13.528$

$S_\eta = \sum A_i \cdot \xi_i = A_1 \cdot \xi_1 + A_2 \cdot \xi_2 - A_3 \cdot \xi_3$

$S_\eta = 12 \cdot 2 + 2 \cdot 0.666 - 3.14 \cdot 3 = 15.912$

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Postupak određivanja glavnih momenata inercije može se sprovoditi:

- izračunavanje težišnih momenata inercije za svaku površinu pa direktnom primenom Štajnerove teoreme odrediti momente inercije za težišne ose površine sabiranjem sopstvenih i položajnih momenata inercije za paralelne ose

$$I_x = I_{x1} + y_1^2 \cdot A_1 + I_{x2} + y_2^2 A_2 - (I_{x3} + y_3^2 A_3)$$

- Izračunavanje momenata inercije sabiranjem ivičnih momenata inercije pa primenom Štajnerove teoreme za poznate paralelne ose oduzimanje položajnog momenta dobiti sopstveni momenti inercije – za težišne ose složene površine

$$I_x = I_\xi - (y_c)^2 \cdot A$$

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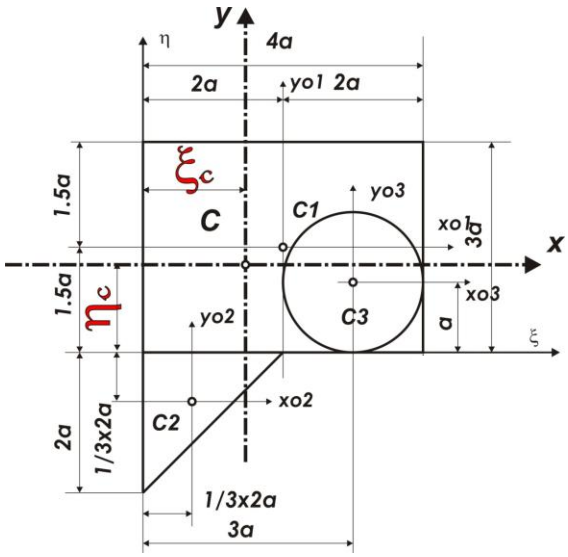
direktnom primrnom Štajnerove teoreme na težišne ose

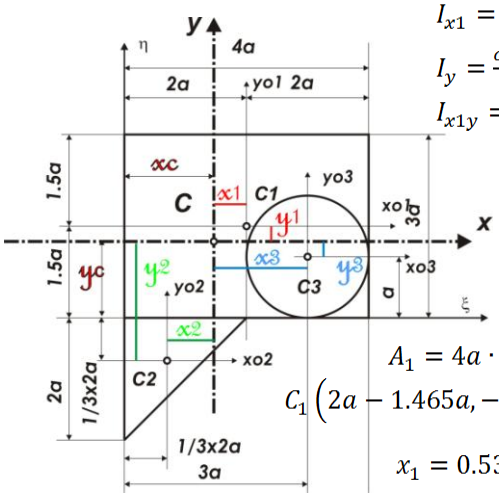


$$\eta_c = \frac{13.528}{10.86} = 1.245cm$$

$$\xi_c = \frac{15.912}{10.86} = 1.465\ cm$$

10

direktnom primrnom Štajnerove teoreme na težišne ose



$$I_{x1} = \frac{ch^3}{12} = \frac{4a\cdot(3a)^3}{12} = \frac{108a^4}{12} = 9cm^4$$

$$I_y = \frac{c^3h}{12} = \frac{64a^3\cdot 3a}{12} = \frac{192a^4}{12} = \frac{16a^4}{3} = 16cm^4$$

$$I_{x1y} = 0$$

$$A_1 = 4a \cdot 3a = 12a^2 = 12cm^2,$$

$$C_1\left(2a - 1.465a, -\frac{3a}{2} + 1.245a\right) = (0.535, -0.225)$$

$$x_1 = 0.535\ cm$$

$$y_1 = -0.225cm$$

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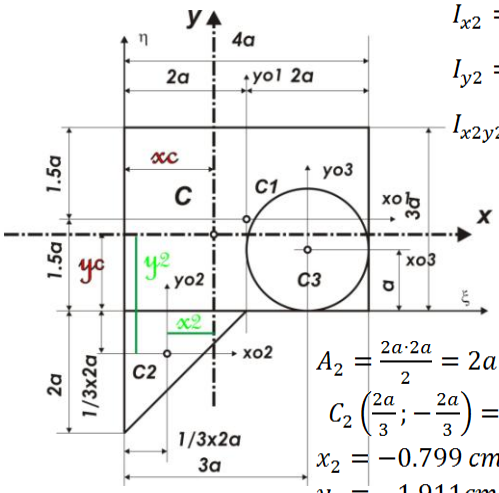
direktnom primrnom Štajnerove teoreme na težišne ose



$$I_{x2} = \frac{ch^3}{36} = \frac{2a\cdot(2a)^3}{36} = \frac{108a^4}{36} = 3cm^4$$

$$I_{y2} = \frac{hc^3}{36} = \frac{8a^3\cdot 2a}{36} = \frac{16a^4}{36} = 0.444cm^4$$

$$I_{x2y2} = \frac{h^2c^2}{72} = \frac{(2a)^2\cdot(2a)^2}{72} = 0.222cm^4$$

$$A_2 = \frac{2a\cdot 2a}{2} = 2a^2 = 2cm^2,$$

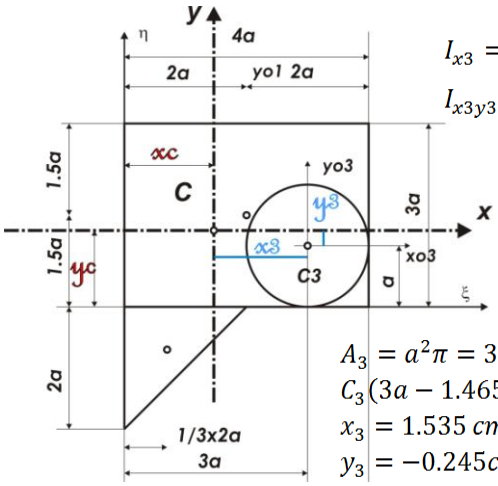
$$C_2\left(\frac{2a}{3}; -\frac{2a}{3}\right) = (-1.465 + 0.666; -0.666 - 1.245)$$

$$x_2 = -0.799\ cm$$

$$y_2 = -1.911cm$$

12

direktnom primrnom Štajnerove teoreme na težišne ose



$$I_{x3} = I_{y3} = \frac{d^4 \cdot \pi}{64} = \frac{R^4 \cdot \pi}{4} = 0.785\ cm^4$$

$$I_{x3y3} = 0$$

$$A_3 = a^2\pi = 3.14\ cm^2$$

$$C_3(3a - 1.465a, 1.245a - a) = (1.535, -0.245)$$

$$x_3 = 1.535\ cm$$

$$y_3 = -0.245 cm$$

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direktnom primrnom Štajnerove teoreme na težišne ose

$$I_x = I_{x1} + y_1^2 \cdot A_1 + I_{x2} + y_2^2 A_2 - (I_{x3} + y_3^2 A_3)$$

$$I_x = 9 + 0.254^2 \cdot 12 + 0.444 + 1.912^2 \cdot 2 - (1.047 + 0.245^2 \cdot \pi)$$

$$I_x = 16.294\ cm^4$$

$$I_y = I_{y1} + x_1^2 \cdot A_1 + I_{y2} + x_2^2 A_2 - \left(I_{y3} + x_3^2 A_3\right)$$

$$I_y = 16 + 0.535^2 \cdot 12 + 0.444 + 0.799^2 \quad -(1.047 + 1.535^2 \cdot \pi)$$

$$I_y = 12.706\ cm^4$$

$$I_{xy} = I_{x1y1} + x_1 \cdot y_1 \cdot A_1 + I_{x2y2} + x_2 \cdot y_2 \cdot A_2 - \left(I_{x3y3} + x_3 \cdot y_3 \cdot A_2\right)$$

$$I_{xy} = 0 - 0.535 \cdot 0.254 \cdot 12 - 0.222 - \ 0{,}799 \cdot 1.912 \cdot 2 - 0 - 1.535 \cdot 0.245 \cdot \pi$$

$$I_{xy} = -6.104\ cm^4$$

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direktnom primrnom Štajnerove teoreme na težišne ose

$I_x = 16.294\ cm^4 \qquad I_y = 12.706\ cm^4 \qquad I_{xy} = -6.104\ cm^4$

$tan\,2\alpha = \frac{-2I_{xy}}{I_x - I_y} = \frac{2 \cdot 6.104}{16.575 - 12.765} = 3.402$

$2 \cdot \alpha = artg 3.402 = 73.621 \rightarrow \alpha = 36.801°$

$I_{12} = \frac{1}{2}(I_x + I_y) \pm \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$I_{12} = \frac{1}{2}(16.294 + 12.706) \pm \frac{1}{2}\sqrt{(16.294 - 12.706)^2 + 4 \cdot 6.104^2}$

$I_{12} = \frac{1}{2}29 \pm \frac{1}{2}12{,}724 = 14.5 \pm 6.36$

$I_1 = 20.862\ cm^4$

$I_2 = 8.138\ cm^4$

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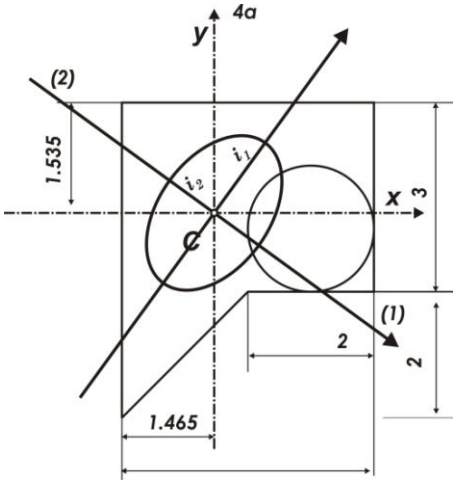
direktnom primrnom Štajnerove teoreme na težišne ose



$I_1 = 20.862\ cm^4$

$I_2 = 8.138\ cm^4$

$i_1 = \sqrt{\frac{I_1}{A}} = \sqrt{\frac{20.862}{10.858}} = 1.386 cm$

$i_2 = \sqrt{\frac{I_2}{A}} = \sqrt{\frac{8.138}{10.858}} = 0.865\ cm$

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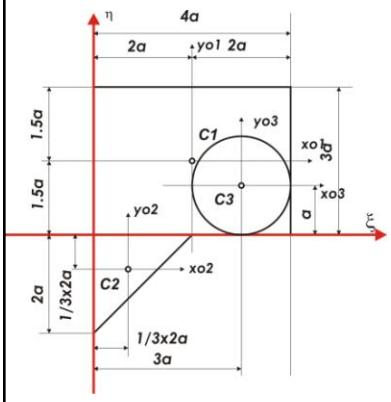
određivanje momenata inercije za odabrane ivične ose



$$I_{\xi 1} = \frac{ch^3}{3} = \frac{4a \cdot (3a)^3}{3} = 36a^4 = 36cm^4$$

$$I_{\eta 1} = \frac{ch^3}{3} = \frac{3a \cdot (4a)^3}{3} = 64a^4 = 64cm^4$$

$$I_{\xi 1 \eta 1} = 0$$

$$I_{\xi 2} = \frac{ch^3}{12} = \frac{2a(2a)^3}{12} = \frac{16a^4}{12}$$

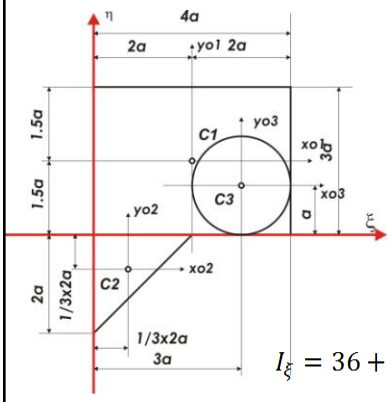
$$I_{\eta 2} = \frac{hc^3}{12} = \frac{2a(2a)^3}{12} = \frac{16a^4}{12}$$

$$I_{\xi 2 \eta 2} = -\frac{h^2 c^2}{12} = -\frac{(2a)^2 (2a)^2}{12} = -\frac{16a^4}{24}$$

$$I_{x3} = I_{y3} = \frac{d^4 \cdot \pi}{64} = \frac{R^4 \cdot \pi}{4} = 0.785\ cm^4$$

$$I_{x3y3} = 0$$

određivanje momenata inercije za odabrane ose



$$I_{\xi 1} = \frac{ch^3}{3} = \frac{4a \cdot (3a)^3}{3} = 36a^4 = 36cm^4$$

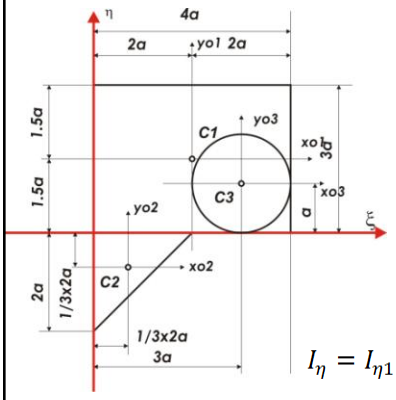
$$I_{\xi 2} = \frac{ch^3}{12} = \frac{2a(2a)^3}{12} = \frac{16a^4}{12}$$

$$I_{x3} = I_{y3} = \frac{d^4 \cdot \pi}{64} = \frac{R^4 \cdot \pi}{4} = 0.785\ cm^4$$

$$I_{\xi} = I_{\xi 1} + I_{\xi 2} - [I_{x3} + a^2 \cdot A_3]$$

$$I_{\xi} = 36 + 1.333 - [0.785 + 1 \cdot 3.14] = 33.408\ cm^4$$

određivanje momenata inercije za odabrane ose



$$I_{\eta 1} = \frac{ch^3}{3} = \frac{3a\cdot(4a)^3}{3} = 64a^4 = 64cm^4$$

$$I_{\eta 2} = \frac{hc^3}{12} = \frac{2a(2a)^3}{12} = \frac{16a^4}{12}$$

$$I_{x3} = I_{y3} = \frac{d^4\cdot\pi}{64} = \frac{R^4\cdot\pi}{4} = 0.785\ cm^4$$

$$I_{x3y3} = 0$$

$$I_\eta = I_{\eta 1} + I_{\eta 1} - \left[I_{y3} + (3a)^2\cdot A_3\right]$$

$$I_\eta = 64 + 1.333 - [0.785 + 9\cdot 3.14] = 36.073\ cm^4$$

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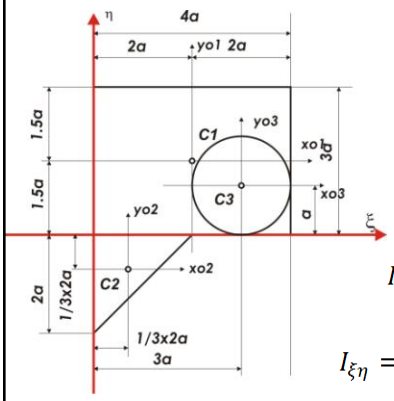
određivanje momenata inercije za odabrane ose



$$I_{\xi 1\eta 1} = 0$$

$$I_{\xi 2\eta 2} = -\frac{h^2c^2}{12} = -\frac{(2a)^2(2a)^2}{12} = -\frac{16a^4}{24}$$

$$I_{x3y3} = 0$$

$$I_{\xi\eta} = I_{\xi 1\eta 1} + I_{\xi 2\eta 2} - \left[\, I_{x3y3} + 3a\cdot a\cdot A_3\right]$$

$$I_{\xi\eta} = 0 - 0.666 - [\,0 + 3\cdot 3.14] = -10.086cm^4$$

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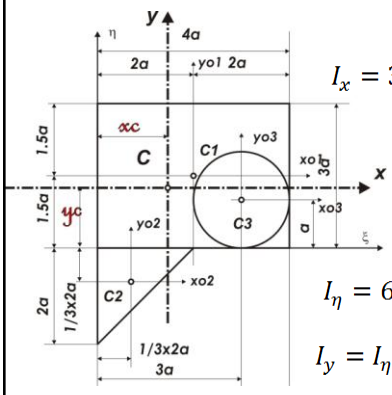
Momenti inercije za težišne ose

$$I_\xi = 36 + 1.333 - [0.785 + 1 \cdot 3.14] = 33.408\ cm^4$$



$$I_x = I_\xi - (y_c)^2 \cdot A$$

$$I_x = 33.408 - (1.245)^2 \cdot 10.86 = 16.575\ cm^4$$

$$I_\eta = 64 + 1.333 - [0.785 + 3 \cdot 3.14] = 55.128\ cm^4$$

$$I_y = I_\eta - (x_c)^2 \cdot A$$

$$I_y = 55.128 - (1.465)^2 \cdot 10.86 = 31.819\ cm^4$$

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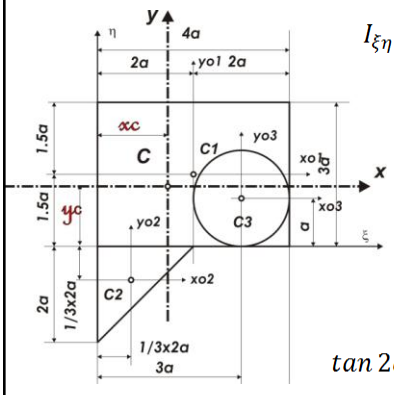
Momenti inercije za težišne ose



$$I_{\xi\eta} = 36 + 0.666 - [\,0 + 3 \cdot 3.14] = 25.908\ cm^4$$

$$I_{xy} = I_{\xi\eta} - x_C \cdot y_c \cdot A$$

$$I_{xy} = 25.908 - 1.465 \cdot 1.245 \cdot 10.858$$

$$I_{xy} = -6.104\ cm^4$$

$$tan\ 2\alpha = \frac{-2I_{xy}}{I_x - I_y} = \frac{2 \cdot 6.104}{16.575 - 12.765} = 3.402$$

$$2 \cdot \alpha = artg 3.402 = 73.621 \rightarrow \alpha = 36.801°$$

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određivanje momenata inercije za glavne ose

$I_x = 16.294\ cm^4 \qquad I_y = 12.706\ cm^4 \qquad I_{xy} = -6.104\ cm^4$

$$tan\,2\alpha = \frac{-2I_{xy}}{I_x - I_y} = \frac{2 \cdot 6.104}{16.575 - 12.765} = 3.402$$

$$2 \cdot \alpha = artg 3.402 = 73.621 \rightarrow \alpha = 36.801°$$

$$I_{12} = \frac{1}{2}(I_x + I_y) \pm \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

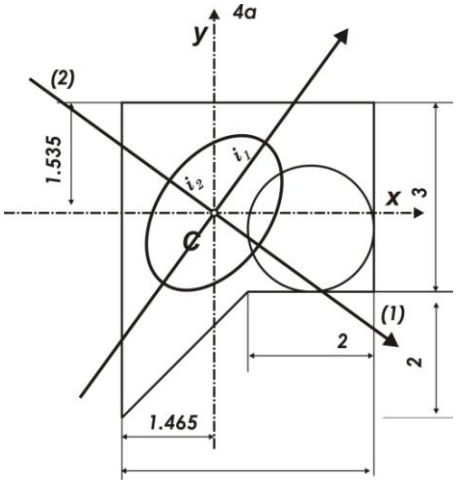
$$I_{12} = \frac{1}{2}(16.294 + 12.706) \pm \frac{1}{2}\sqrt{(16.294 - 12.706)^2 + 4 \cdot 6.104^2}$$

$$I_{12} = \frac{1}{2}29 \pm \frac{1}{2}12{,}724 = 14.5 \pm 6.36$$

$I_1 = 20.862\ cm^4$

$I_2 = 8.138\ cm^4$

određivanje momenata inercije za glavne ose



$I_1 = 20.862\ cm^4$

$I_2 = 8.138\ cm^4$

$$i_1 = \sqrt{\frac{I_1}{A}} = \sqrt{\frac{20.862}{10.858}} = 1.386cm$$

$$i_2 = \sqrt{\frac{I_2}{A}} = \sqrt{\frac{8.138}{10.858}} = 0.865\ cm$$